4 Lasing

In the following we will assume the small signal gain to be uniform in space, and neglect any transverse stucture in saturation due to beam shape. To reach lasing threshhold the roundtrip gain must exceed the resonator losses. Including all distributed losses into the discrete reflection coefficients of the mirrors the lasing threshold condition is

$$e^{2g_0L}R_1R_2 > 1 \qquad \text{or} \qquad g_0L + \ln\sqrt{R_1R_2} > 0 \tag{1}$$

where L is the length of the gain region.

4.1 Power output

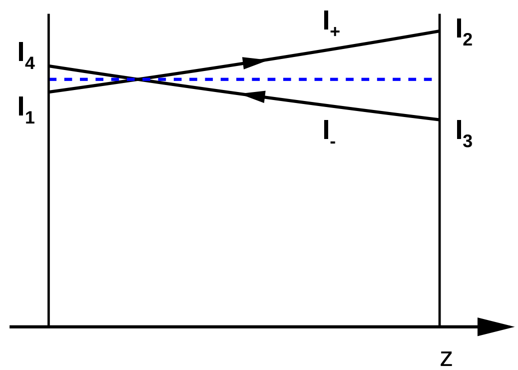
We treat the homogeneous saturation first and follow the analysis of Rigrod. The right going beam inside the resonator gets the name I_+ and the leftgoing I_-. The coupled differential equations describing the beam growth are

$$\frac{dI_+}{I_+} = -\frac{dI_-}{I_-} = \frac{g_0(\omega)}{1+\frac{I_+ + I_-}{I_S}} \tag{2}$$

The left part of eq. 2 leads to the condition that the product of the beam intensities is fixed,

$$I_+I_- = I_0^2 \tag{3}$$

where I_0 is an integration constant, independent of z. Assigning the names I_m to the beam intensities at the mirrors according to the figure the integration of the second part of eq. 2 leads to the condition



$$\ln\frac{I_2}{I_1} + \frac{I_2 - I_1}{I_S} - \frac{I_3}{I_S}\left(1 - \frac{I_2}{I_1}\right) = g_0(\omega)L \tag{4}$$

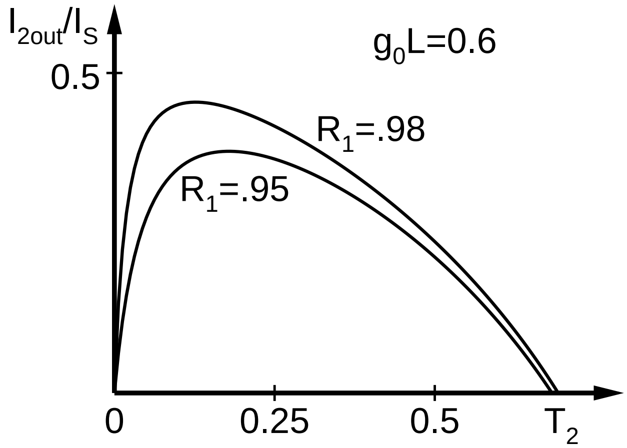
Imposing the boundary conditions $I_1 = R_1I_4$ and $I_3 = R_1I_2$ leads to the solution for the ratio I_2/I_S

$$\frac{I_2}{I_S} = \frac{1}{[1+\sqrt{R_2/R_1}]}\;\frac{(g_0(\omega)L + \ln\sqrt{R_1R_2}\,)}{[1-\sqrt{R_1R_2}]} \tag{5}$$

This expression is of course only valid above threshold. The first fraction on the right side is only slowly varying with the reflection coefficients whereas the second fraction is sensitive to the coefficients product near threshold and for high values of the product.

For a lossless mirror the laser output through mirror 2 is given with

$$I_{2out} = I_2T_2 = I_2(1-R_2) = I_S\,\frac{(1-R_2)}{[1+\sqrt{R_2/R_1}]}\;\frac{(g_0(\omega)L + \ln\sqrt{R_1R_2}\,)}{[1-\sqrt{R_1R_2}]} \tag{6}$$



Rough frequency selection is done with a dispersive element like a reflection grating which adds frequency dependent losses to the system. The laser will pick the cavity resonance that has the highest difference between gain and losses. The saturation will then see to that all other resonances end up with a negative roundtrip gain, and thereby securing single mode oscillation.

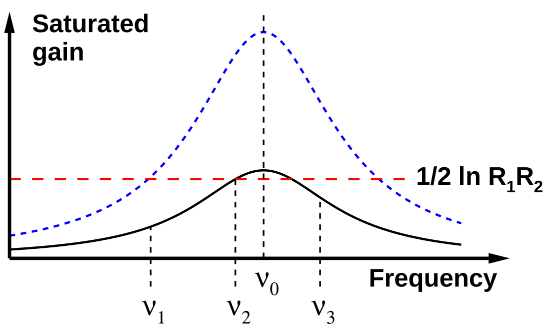
Fine frequency tuning is then done by small changes in cavity length.

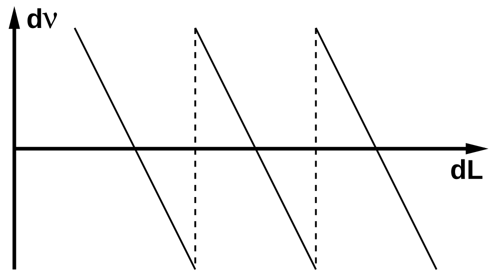
$$\nu_{pnm} = \frac{c}{2L}\left[p+(1+n+m)\frac{(\phi_2-\phi_1)}{\pi}\right] \tag{7}$$

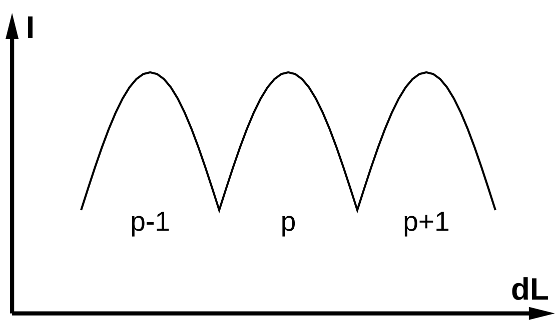
$$p \simeq \frac{L}{\lambda/2} \tag{8}$$

$$d\nu_{pnm} \simeq -\frac{c}{2L}\,p\,\frac{dL}{L} \simeq -\frac{c}{2L}\,\frac{dL}{\lambda/2} \tag{9}$$

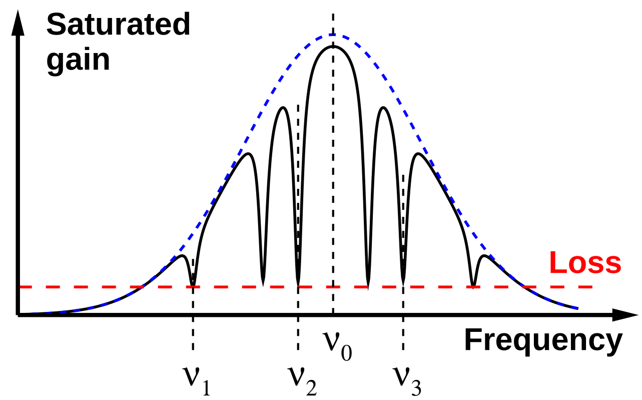
The tuning range is limited to $\pm\frac{c}{4L}$ since beyond this range the neighbouring longitudinal mode has more favorable gain closer to linecenter. To suppress the onset of higher order transverse modes an iris is often placed in the resonator. The diameter is selected to cause higher loss to the transverse modes than the fundamental.







The inhomogeneous case is more complicated for several reasons. Saturation leads to local holeburning into the gain profile and mode competition is thereby limited. The laser can swing on several axial modes at the same time. As the beam travels in both directions inside the cavity each oscillation frequency also creates a mirrorhole symmetrically placed at the other side of linecenter. Cavity length tuning will lead to hole collisions. Hole overlap generates mode competition and reduction in power output.



The system displayed in the figure is oscillating on the 3 frequencies ν_1, ν_2 and ν_3 and digs 6 holes into the profile including the mirror holes. The relative intensities are given by hole area ratioes. Higher order transverse modes are assumed suppressed with mode selective losses.