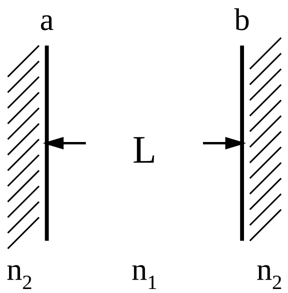
1 Fabry-Perot geislahólf

1.1 Full speglandi planfletir



$$R_a = R_b = 1$$

$$r = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1 - n' - i\kappa'}{1 + n' + i\kappa'} \longrightarrow -1 \qquad \text{ef } n' \to \infty \quad \text{eða} \quad \kappa' \to \infty$$

Planbylgja speglast fram og til baka

$$E = E_0 e^{-i(\omega t - kz)} + rE_0 e^{-i(\omega t + kz)} = E_0 e^{-i\omega t} \left(e^{ikz} + re^{-ikz}\right)$$

Randskilyrði: $E_{inn} + E_{spegl} = E_{refr} = 0$

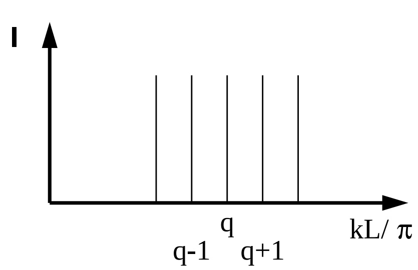
$$\Longrightarrow \quad E(z=0) = E(z=L) = 0$$

$$z = 0: \qquad 1 + r = 0 \qquad \Longrightarrow \qquad r = -1$$

$$z = L: \qquad e^{-ikL} = e^{ikL} \qquad \Longrightarrow \qquad kL = q\pi$$

$$L = q\,\frac{\lambda}{2} \qquad \text{eða} \qquad \nu = q\,\frac{c}{2L}$$

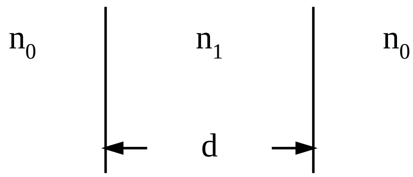
$$\Delta\nu = \nu_{q+1} - \nu_q = \frac{c}{2L} \qquad \text{free spectral range}$$



Ekkert tap

$\Longrightarrow$ delta fn. resonansar

1.2 Etalon $\quad R < 1$



Fresnel:

$$r_{01} = -r_{10}$$
$$n_1 t_{01} = n_0 t_{10}$$
$$R_{01} = R_{10} = R$$
$$T_{01} = \tfrac{n_1}{n_0} t_{01}^2 = t_{01} t_{10}$$

$$r_{etalon} = \frac{E_{sp}}{E_0} = r_{01} + t_{01} r_{10} t_{10} e^{2ik_1 d} + t_{01} r_{10}^3 t_{10} e^{4ik_1 d} + \cdots$$

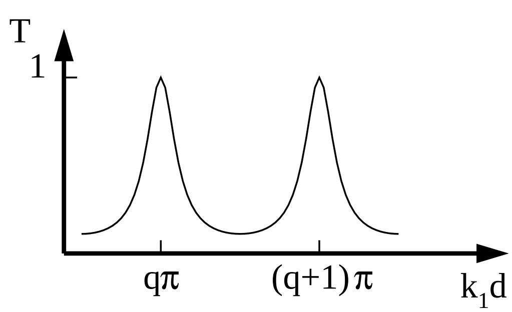
$$= r_{01} \left[1 - t_{01} t_{10} e^{2ik_1 d} \{ 1 + r_{10}^2 e^{2ik_1 d} + \cdots \} \right]$$

$$= r_{01} \left[1 - \frac{T e^{2ik_1 d}}{1 - R e^{2ik_1 d}} \right] = r_{01} \frac{1 - e^{2ik_1 d}}{1 - R e^{2ik_1 d}}$$

$$R_{etalon} = \frac{R\,[(1-\cos 2k_1 d)^2 + \sin^2 2k_1 d]}{(1 - R\cos 2k_1 d)^2 + R^2 \sin^2 2k_1 d}$$

$$= \frac{2R(1-\cos 2k_1 d)}{1 - 2R\cos 2k_1 d + R^2} = \frac{4R\sin^2 k_1 d}{(1-R)^2 + 4R\sin^2 k_1 d}$$

$$T_{etalon} = 1 - R_{etalon} = \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2 k_1 d} = \frac{1}{1 + \frac{4R\sin^2 k_1 d}{(1-R)^2}}$$



$$T_{max} = 1$$
$$T_{min} = \left(\tfrac{1-R}{1+R}\right)^2$$

Hálflínubreidd: $\quad R \gg \frac{\sqrt{2}-1}{\sqrt{2}+1} \simeq 17\%$

$$T_{etalon} = 1/2 \implies (1-R)^2 = 4R\sin^2 k_1 d \qquad \sin k_1 d = \frac{1-R}{2\sqrt{R}} \simeq \phi_{1/2}$$

Finesse: $\quad F = \dfrac{\pi}{2\phi_{1/2}} = \dfrac{\pi\sqrt{R}}{1-R}$

Bil milli resonanstoppa ræðst eins og áður af skilyrðinu

$$k_1 d = \frac{2\pi n_1 d}{\lambda_0} = q\pi$$

$$\nu = q\frac{c}{2n_1 d} \qquad \text{og} \qquad \Delta\nu = \frac{c}{2n_1 d}$$

Topparnir hafa fengið endanlega breidd $2\phi_{1/2}$ (FWHM) þar sem orka lekur úr geislaholinu ($R < 1$).

Almennt gildir um einföld skil milli brotstuðla án ísogs að $1 = R + T$ Hæsti gegnskinsstuðull fyrir etalon er $(T_{etalon})_{max} = 1.$ Innan etalonsins er styrkurinn I_1.

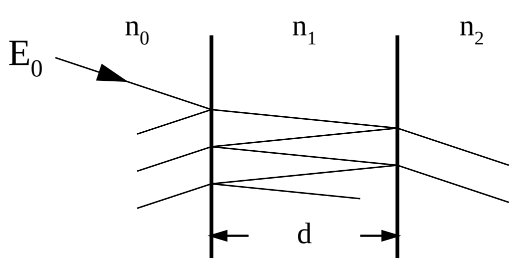
$$I_1 = \frac{I_0}{T} = \frac{I_0}{1-R}$$

Fyrir háa Finessu verður þessi styrkur miklu stærri en styrkur innfallsgeislans. Lægsti styrkur verður

$$I_1 = \frac{1}{T}\left(\frac{1-R}{1+R}\right)^2 I_0 = \frac{1-R}{(1+R)^2}\, I_0$$

Etalon má því nýta bæði sem tíðnisíu og styrkmagnara.

1.3 Þunnar húðir



$$\boxed{\begin{array}{cc} r_{ij} = -r_{ji} & R_{ij} = |r_{ij}|^2 \\ n_j t_{ij} = n_i t_{ji} & T_{ij} = \frac{n_j}{n_i}|t_{ij}|^2 = t_{ij}t_{ji} \\ 1 = R_\alpha + T_\alpha & 2\phi = 2k_1 d \end{array}}$$

Hér erum við komin einu flækjustigi ofar en með etalon þar sem samhverfan er horfin með því að hægra megin húðar er $n_2 \neq n_0$. Við söfnum saman eins og áður spegluðum hlutgeislum eftir mismunandi margar ferðir fram og til baka innan húðar.

$$E_0' = E_0\,[\, r_{01} + t_{01}r_{12}t_{10}e^{2i\phi} + t_{01}r_{12}^2 r_{10}t_{10}e^{4i\phi} + \cdots \,]$$

$$= E_0\,[\, r_{01} + T_{01}r_{12}e^{2i\phi}\,\{\, 1 + r_{12}r_{10}e^{2i\phi} + \cdots \,\}\,]$$

$$= E_0\left[\, r_{01} + \frac{T_{01}r_{12}e^{2i\phi}}{1 - r_{12}r_{10}e^{2i\phi}}\,\right] = E_0\,\frac{-r_{10} + r_{12}(R_{01}+T_{01})e^{2i\phi}}{1 - r_{12}r_{10}e^{2i\phi}}$$

$$= E_0 \, \frac{r_{01} \, + \, r_{12} e^{2i\phi}}{1 \, + \, r_{12} r_{01} e^{2i\phi}}$$

$$\Longrightarrow \qquad r \, = \, \frac{E_0'}{E_0} \, = \, \frac{r_{01} \, + \, r_{12} e^{2i\phi}}{1 \, + \, r_{01} r_{12} e^{2i\phi}} \qquad\qquad r_{ij} \, = \, \frac{n_i - n_j}{n_i + n_j}$$

Á svipaðan hátt getum við dregið saman framlögin sem mynda gegnskinsstuðulinn t fyrir húðina. Speglun og gegnskini húðar fylgir almennt fasasnúningur meðan einföld skil gáfu í hæsta lagi formerkjaskipti.

Á þessu stigi er meinhollt að velta fyrir sér hvernig við getum beitt þessum aðferðum á kerfi með mörgum lögum mismunandi efna.

Skoðum nú sértilfellið $\phi = \pi/2$ í jöfnunni sem lýsir r hér að ofan. $\Longrightarrow \; d = \dfrac{\lambda}{4n_1}$

$$r \, = \, \frac{(n_0 - n_1)(n_1 + n_2) \, - \, (n_1 - n_2)(n_0 + n_1)}{(n_0 + n_1)(n_1 + n_2) \, - \, (n_0 - n_1)(n_1 - n_2)} \, = \, \frac{n_0 n_2 - n_1^2}{n_0 n_2 + n_1^2}$$

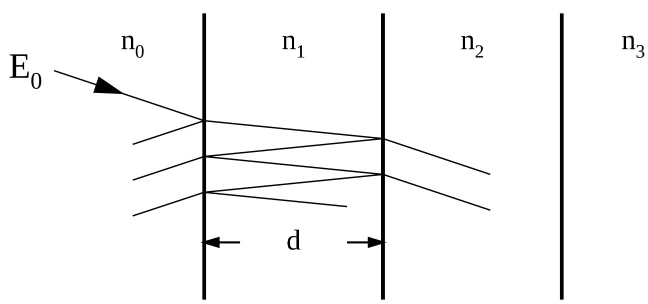
$$\Longrightarrow \qquad r = 0 \;\; \text{ef} \;\; n_1^2 = n_0 n_2 \qquad \text{Antireflexhúð}$$

Almennt gildir: $|r| \, < \, |r_{02}|$ ef $n_0 < n_1 < n_2$

$|r| \, > \, |r_{02}|$ ef $n_1 > n_2$

Setjum $\phi = \pi \Longrightarrow \quad r \, = \, \dfrac{n_0 - n_2}{n_0 + n_2}$ óháð n_1, sami speglunarstuðull og fæst án húðar.

Til að marka slóð við greiningu á fleiri lögum húða skoðum við tveggja laga kerfi eins og sýnt er á myndinni hér fyrir neðan.



Jöfnuna fyrir speglunarstuðul einfalds lags umritum við lítillega

$$r \, = \, \frac{r_{01} \, + \, r_{12}^* e^{2i\phi_1}}{1 \, + \, r_{01} r_{12}^* e^{2i\phi_1}}$$

þar sem stærðin r^*_{12} stendur fyrir speglunarstuðul skilflatarins 1,2 og alls sem er hægramegin við hann á myndinni.

$$r^*_{12} \;=\; \frac{r_{12} \;+\; r_{23}e^{2i\phi_2}}{1 \;+\; r_{12}r_{23}e^{2i\phi_2}}$$

$\lambda/4$ lög $\quad\Longrightarrow\quad e^{2i\phi_x} \;=\; -1$

$$r^*_{12} \;=\; \frac{r_{12} \;-\; r_{23}}{1 \;-\; r_{12}r_{23}}$$

$$r \;=\; \frac{r_{01}-r_{12}+r_{23}-r_{01}r_{12}r_{23}}{1-r_{12}r_{23}-r_{01}r_{12}+r_{01}r_{23}} \;=\; \frac{n_0n_2^2-n_1^2n_3}{n_0n_2^2+n_1^2n_3}$$

AR fyrir $n_2 = n_1\sqrt{n_3}$

Skilgreinum kontrast-stuðul α með $n_1 = \alpha n_2$ og setjum $n_0 = 1$

$$\Longrightarrow \quad r = \frac{1-\alpha^2 n_3}{1+\alpha^2 n_3} \quad \Longrightarrow \quad \text{Hár speglunarstuðull f.} \quad \alpha > 1$$